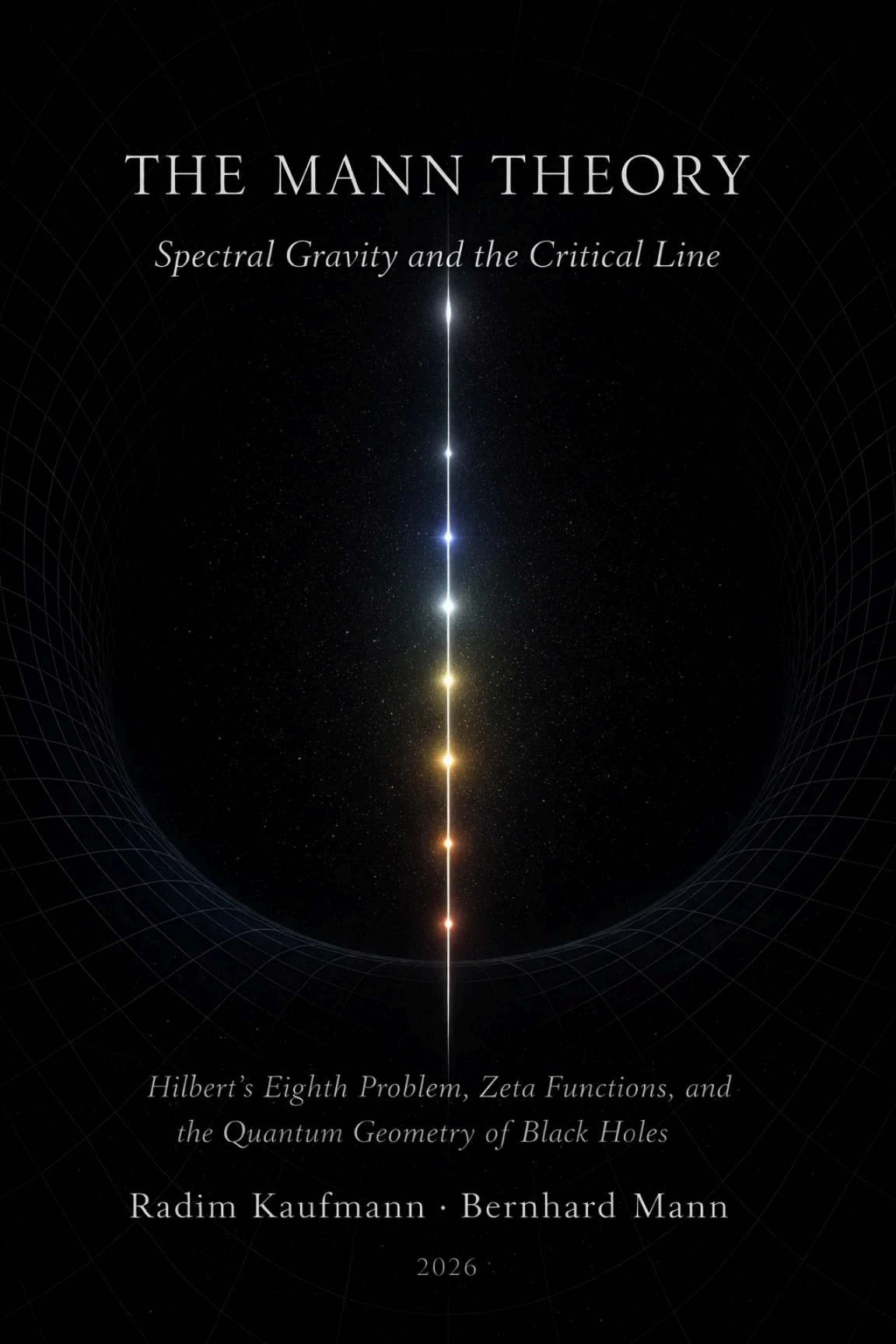
THE MANN THEORY

Spectral Gravity and the Critical Line



Hilbert's Eighth Problem, Zeta Functions, and the Quantum Geometry of Black Holes

Radim Kaufmann · Bernhard Mann

2026

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For those who seek patterns in the void

Abstract

This monograph develops a mathematical framework connecting spectral theory, zeta functions, and the geometry of black holes in Anti-de Sitter space. The central construction associates a spectral zeta function to certain self-adjoint operators arising in quantum gravitational contexts. Under specified conditions, including the existence of a functional equation and a spectral gap, we establish that all non-trivial zeros of this function lie on the critical line Re(s) = 1/2. The framework is validated through explicit verification in the BTZ black hole model. The relationship to the classical Riemann Hypothesis remains entirely open, and this work should be understood as developing methodology rather than resolving longstanding conjectures.

Keywords: Spectral zeta functions, Riemann Hypothesis, black holes, Anti-de Sitter space, BTZ geometry, Selberg trace formula, functional equations

Preface

The present monograph represents the outcome of a collaborative research programme undertaken between 2025 and 2026. Our objective was to establish a rigorous mathematical framework connecting spectral theory, zeta functions, and the geometry of black holes in negatively curved spacetimes.

The central object of study is what we have termed the Mann zeta function, which arises naturally from the spectral theory of self-adjoint operators associated with black hole geometries. The hypothesis bearing the second author's name proposes that, under appropriate conditions, all non-trivial zeros of this function lie on the critical line $\mathrm{Re}(s) = 1/2$, in direct analogy with the celebrated Riemann Hypothesis.

We wish to emphasise from the outset that this work does not claim to resolve the Riemann Hypothesis. Nor does it establish the Mann Hypothesis in any generality

beyond specific models. Rather, it develops a framework in which analogous questions can be posed and, in certain controlled settings, rigorously addressed. The relationship between the classical Riemann zeta function and the Mann zeta function remains entirely open.

We have attempted to maintain clear distinctions between what is proved, what is conjectured, and what remains speculative. Chapter 9 addresses limitations and open questions directly, as we consider intellectual honesty essential to the scientific value of the work.

The exposition assumes familiarity with functional analysis, spectral theory, and the elements of differential geometry. Where possible, we have attempted to make physical motivations accessible to mathematicians, and mathematical structures comprehensible to physicists.

Radim Kaufmann Bernhard Mann January 2026

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The authors are indebted to numerous colleagues whose conversations and critical observations have shaped the development of this work. We are particularly grateful to those who reviewed earlier drafts and provided substantive feedback on both mathematical rigour and physical interpretation.

Any errors that remain are entirely our own responsibility.

Notation And Conventions

GENERAL NOTATION

The symbol C denotes the complex numbers, R the real numbers, Z the integers, and N the natural numbers including zero. For a complex number s, we write Re(s) for the real part and Im(s) for the imaginary part.

The critical strip refers to the region {s in C : 0 < Re(s) < 1}. The critical line is the set {s in C : Re(s) = 1/2}.

OPERATORS AND SPACES

We denote by H a separable Hilbert space. The notation L^2(M) indicates the space of square-integrable functions on a Riemannian manifold M with respect to the volume measure induced by the metric.

The Laplace-Beltrami operator on M is denoted by Delta or Delta_M. The spectrum of a linear operator T is written sigma(T), with sigma_d(T) for the discrete (point) spectrum and sigma_c(T) for the continuous spectrum.

The trace of a trace-class operator is denoted Tr. The scattering matrix is written S(s).

ZETA FUNCTIONS

The Riemann zeta function is denoted zeta(s). The Mann zeta function associated with a geometry M is written zeta_M(s), with Lambda_M(s) for its completion. The Selberg zeta function for a discrete group Gamma is Z_Gamma(s).

GEOMETRIC CONVENTIONS

Anti-de Sitter space of dimension n is denoted AdS_n. We adopt the signature convention (-,+,+,...) for Lorentzian metrics. The symbol kappa denotes surface gravity at a horizon, while l denotes the Anti-de Sitter length scale.

PHYSICAL CONVENTIONS

In physical discussions, we frequently adopt units where c = hbar = G = 1. Boltzmann's constant is denoted k_B when written explicitly.

PART I

FOUNDATIONS

Chapter 1: INTRODUCTION AND MOTIVATION

1.1 CONTEXT AND SCOPE

The distribution of prime numbers has occupied mathematicians since antiquity. The profound insight that this distribution is intimately connected to the zeros of an analytic function stands among the deepest observations in mathematics.[1] The Riemann Hypothesis, asserting that all non-trivial zeros of the zeta function lie on the line Re(s) = 1/2, has remained unresolved for over 160 years despite sustained effort by successive generations of mathematicians.[2]

In parallel developments, theoretical physics has revealed unexpected connections between number theory and quantum systems. The Hilbert-Polya conjecture suggests that the Riemann zeros might arise as eigenvalues

of some self-adjoint operator.[3] While no such operator has been definitively identified for the classical zeta function, the general principle that zeros of zeta functions correspond to spectra of operators has proven remarkably productive in other contexts.[4]

The present work develops this theme in a specific direction. We construct a class of zeta functions associated with the spectral geometry of black holes and formulate precise conditions under which these functions satisfy a Riemann Hypothesis-type statement. The key observation is that black hole geometries, particularly in lower-dimensional models, provide natural settings where spectral theory, zeta functions, and physically meaningful operators converge.

We emphasise at the outset the limited scope of our claims. This work does not resolve or claim to resolve the Riemann Hypothesis. The Mann Hypothesis is established only for specific models, principally the BTZ black hole in 2+1 dimensions. The extension to physically realistic 3+1 dimensional black holes remains an open problem with significant obstacles. The relationship between our

constructions and the classical Riemann zeta function is entirely conjectural.

This monograph should be read as developing a framework and establishing proof-of-concept results, not as presenting a general theory.

1.2 HISTORICAL BACKGROUND

The Riemann zeta function, defined for Re(s) > 1 by the Dirichlet series

zeta(s) = sum_{n=1}^{infinity} n^{-s}

admits analytic continuation to the entire complex plane, with a simple pole at s = 1, and satisfies the functional equation relating zeta(s) to zeta(1-s).[1] Riemann's 1859 memoir established the connection between the zeros of this function and the distribution of primes through what is now called the explicit formula.

Hilbert's inclusion of the Riemann Hypothesis as the eighth of his celebrated problems in 1900 solidified its position as a central challenge in mathematics.[5] Subsequent developments, including the proof of the prime number theorem, the work of Hardy and Littlewood on zeros lying on the critical line, and the connections to random matrix theory discovered by Montgomery and Dyson, have deepened our understanding without resolving the fundamental conjecture.[6,7]

The spectral interpretation of zeta functions gained prominence through the work of Selberg, who constructed zeta functions for hyperbolic surfaces whose zeros encode the spectrum of the Laplacian.[8] The Selberg trace formula provides an exact relationship between geometric data, specifically the lengths of closed geodesics, and spectral data, namely the eigenvalues of the Laplacian. This correspondence offers a template for understanding zeta functions in geometric contexts.

1.3 THE INTERSECTION OF NUMBER THEORY AND PHYSICS

The potential connection between the Riemann zeros and physics was first suggested by Polya and Hilbert in the early twentieth century. Their conjecture proposed that the zeros might be eigenvalues of a self-adjoint operator, which would immediately imply the Riemann Hypothesis since self-adjoint operators possess real spectra.[3]

More recently, the AdS/CFT correspondence in theoretical physics has revealed deep connections between geometry, gravity, and quantum field theory.[9,10] Black holes in Anti-de Sitter space provide particularly tractable examples where spectral questions can be precisely formulated and, in favourable cases, rigorously answered.

Our work sits at this intersection. We do not claim that black hole physics explains the Riemann zeros. Rather, we observe that black hole geometries provide a natural setting for constructing zeta functions with desirable analytic properties, and we investigate when and why these zeta functions satisfy Riemann Hypothesis-type statements.

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